A BRIEF REVIEW OF REGGE CALCULUS IN CLASSICAL NUMERICAL RELATIVITY

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We briefly review past applications of Regge calculus in classical numerical relativity, and then outline a programme for the future development of the field. We briefly describe the success of lattice gravity in constructing initial data for the head-on collision of equal mass black holes, and discuss recent results on the efficacy of Regge calculus in the continuum limit.

It has long been hoped that Regge calculus[1] could provide an exciting and independent tool in numerical relativity.[2] However, it is only in the last decade that this simplicial, lattice-based approach to gravity has begun to realise its potential.[3,4,5] In this brief report we describe recent numerical applications of Regge calculus, and outline a programme which we believe will aid the development of the field.

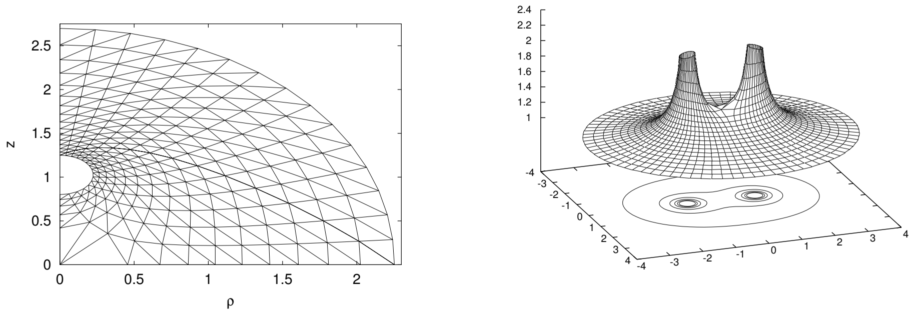
The early development of the Regge approach to lattice gravity focused heavily on highly symmetric toy models for which there are corresponding exact solutions of Einstein's equations. These early studies typically retained only a handful of degrees of freedom in the lattice, and applications included various cosmological spacetimes, together with initial data for single and multiple black holes. We refer the reader to the bibliography and review by Williams and Tuckey[6] for further details on these important pioneering efforts.

We believe the future development of Regge calculus requires a rigorous programme of testing and incremental improvements in both our understanding of the lattice approach and the basic formulation itself. To this end, we propose a series of test-bed applications in which to develop and further our understanding.

The first step of this programme involves a serious examination of Regge calculus in a generic spherically symmetric setting. This allows the comparison of lattice gravity with more standard finite difference techniques for the evolution of isolated black hole spacetimes, and also provides an ideal framework in which to develop a formalism for the inclusion of matter in Regge calculus. Work is currently underway on these issues using a generic simplicial spherically symmetric lattice.

To complement the spherically symmetric work we are also developing a generic axisymmetric code, which will be used to study gravitational radiation on the lattice. Initial data for Brill waves on Minkowski and Schwarzschild "backgrounds" has already been successfully constructed.[7,8] In addition, we have also constructed initial data for the head on collision of two equal mass black holes[5]; see figure 1. Work is currently underway on the time evolution of these sets of initial data.

Lattice gravity has been successfully benchmarked in three-plus-one dimensions on the Kasner cosmology[4]; with the insights gained from the programme outlined above, the ultimate aim is the development of the Regge approach to the point where the relative strengths and weaknesses can be examined in light of more traditional techniques. In the short term, this involves parallelization of the existing



(a) *(b)*

Figure 1. Results from an application of Regge calculus to the construction of Misner initial data for the head on collision of non-rotating, equal mass black holes. We show *(a)* a portion of the lattice, which is based on Čadež coordinates, and *(b)* the conformal factor close to the holes.

$(3+1)$-dimensional code, together with the formulation of suitable lapse and shift conditions. It is also vital in the longer term that techniques are developed to include generic matter terms in the Regge equations.

Finally, we note that the controversy[9,10] over the convergence of Regge calculus to general relativity has been resolved. The claim that Regge calculus failed to converge in the continuum limit was based on the observation that the residual of the Regge equations, when evaluated on carefully interpolated solutions of the Einstein equations, failed to converge. We believe that recent work[11] has provided an explanation of this behaviour which is nevertheless consistent with second order convergence of the solutions of Regge calculus to the continuum.

We believe the future of numerical Regge geometrodynamics is bright.

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